Fluid Mechanics

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Lecture – 18
Problems Solving on Black Board

Very good afternoon to all of you. Today we are going to have fluid kinematics solving some of the problems on the blackboard.

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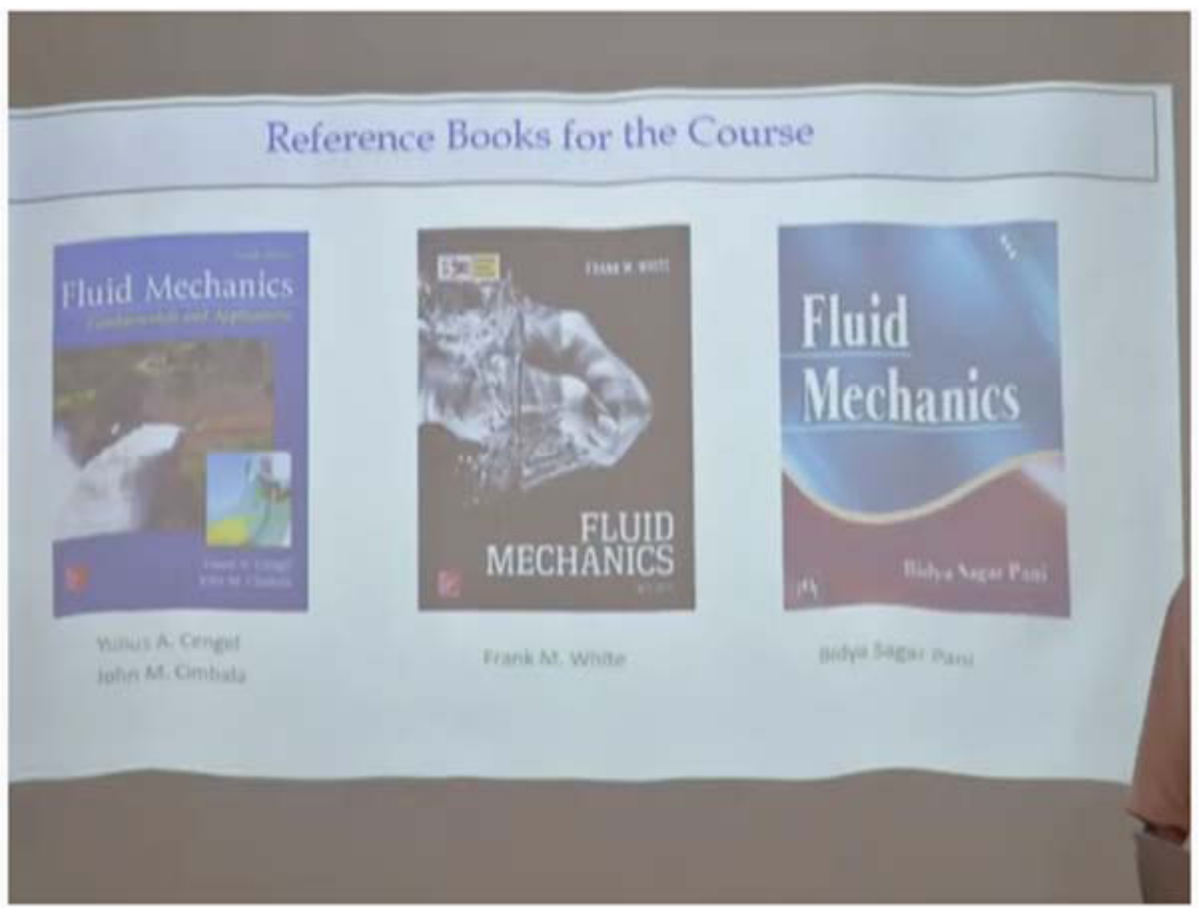

Looking that as I said it earliest we are again having same reference book starting from Cengel, Cimbala, F M White and Bidya Sagar Pani. So my sincere request to you to please look for the book of Cengel, Cimbala book which have which gives lot of illustrations to visualize the fluid flow problems because if would try to understand the fluid kinematics which is very interesting stuff subject.

Beside this solving the problems also we should look at how the flow behavior flow visualizes a technique that is what is very good illustrations are there in Cengel, Cimbala book. So please refer to Cengel, Cimbala book of fluid mechanics and other 2 books as we refer earlier case also.

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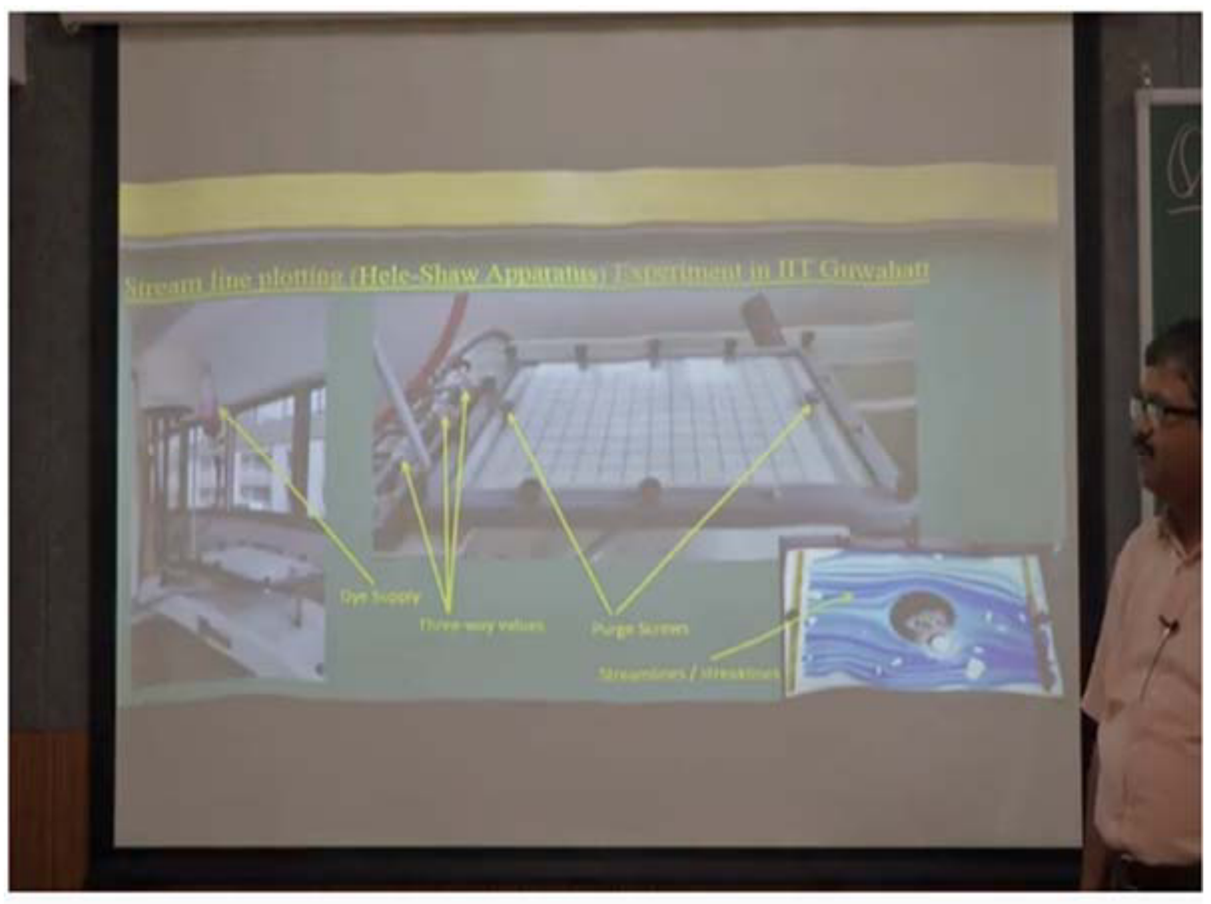

Looking that again I will show it there it could be conduct a very small experiments which is called Heles apparatus. So where we can have an apparatus like these and we can create the streamline pattern, the streakline pattern and pathline patterns using these the small device which is called the Hele-Shaw apparatus. Like for examples if you can look it that if I have the obstruction structures like these and having the flow patterns like these you can see the streamline patterns what is going on near the structures and far away from the structures. So very easy to visualize the flow when you use the apparatus like Hele-Shaw apparatus.

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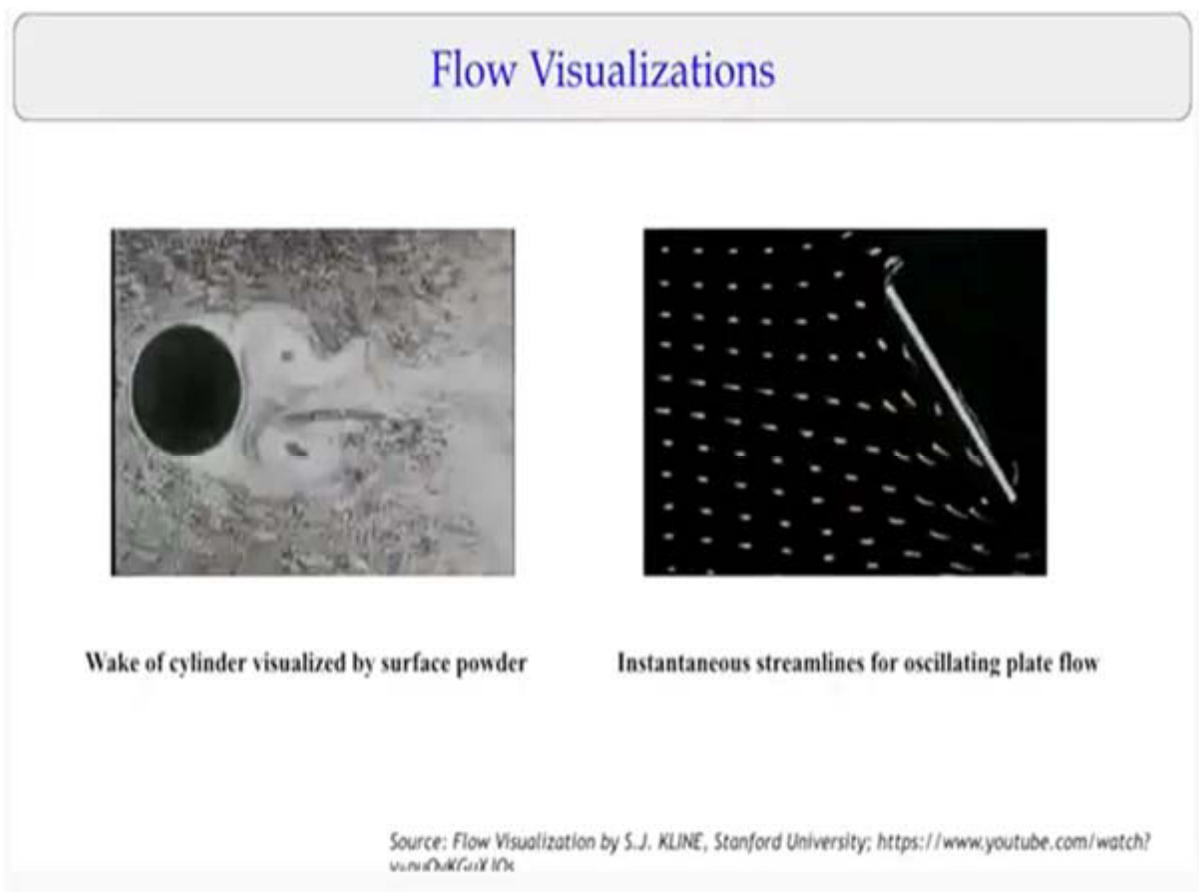

And if you look it very interesting flow visualizations are available in internet. Please refer to look at these flow visualization details, videos what these are available in the internets. Like for examples here it is showing a how the wake formation happens just behind of the cylinders when uniform flow is going on and how the wake formation happens is the very interesting phenomena.

Similar way you can have the oscillating plate and because of the oscillating plate how the streamline patterns are changing with respect to the time, the pathline, the streaks line all you can visualize using this type of video. So my sincere suggestions to you please visualize the flow visualizations by visiting this the YouTube sites you can see that how the flow patterns are changing it.

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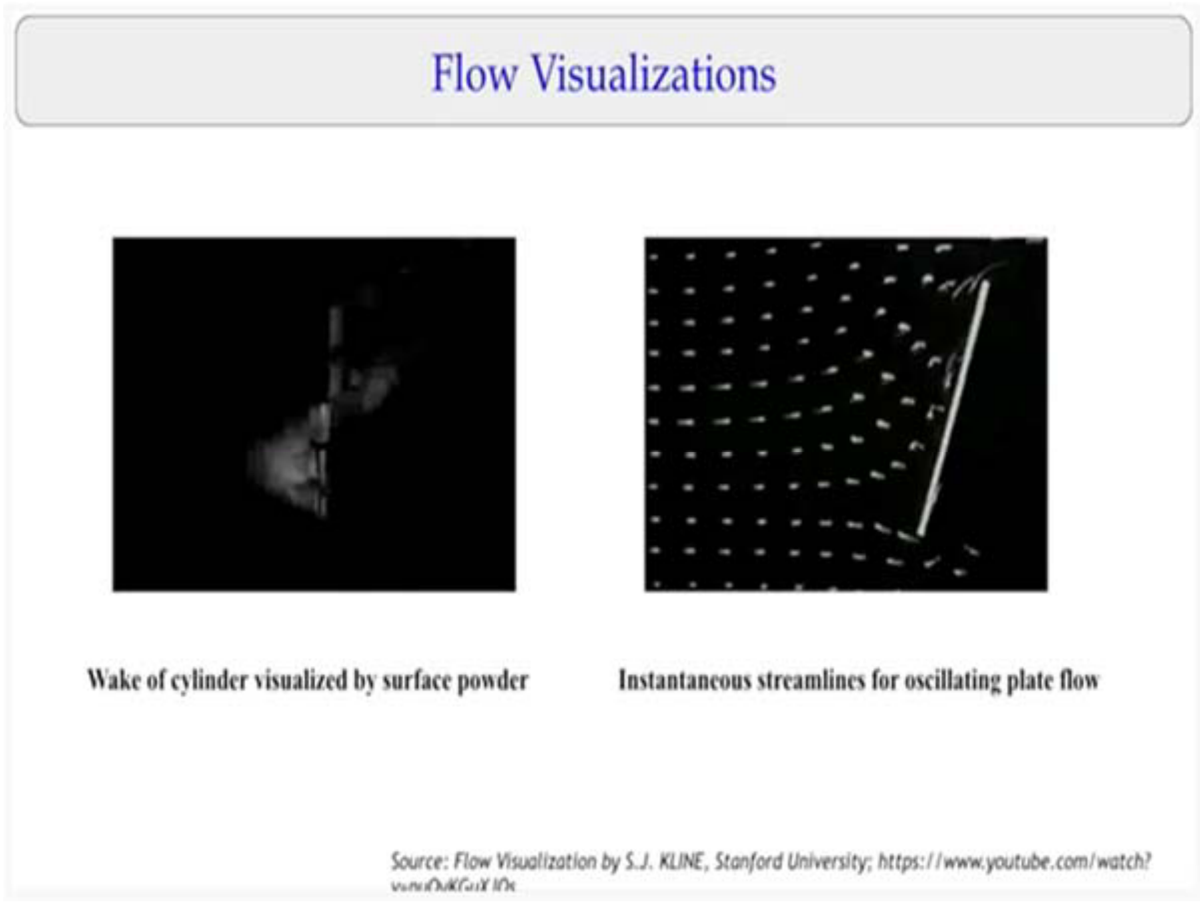

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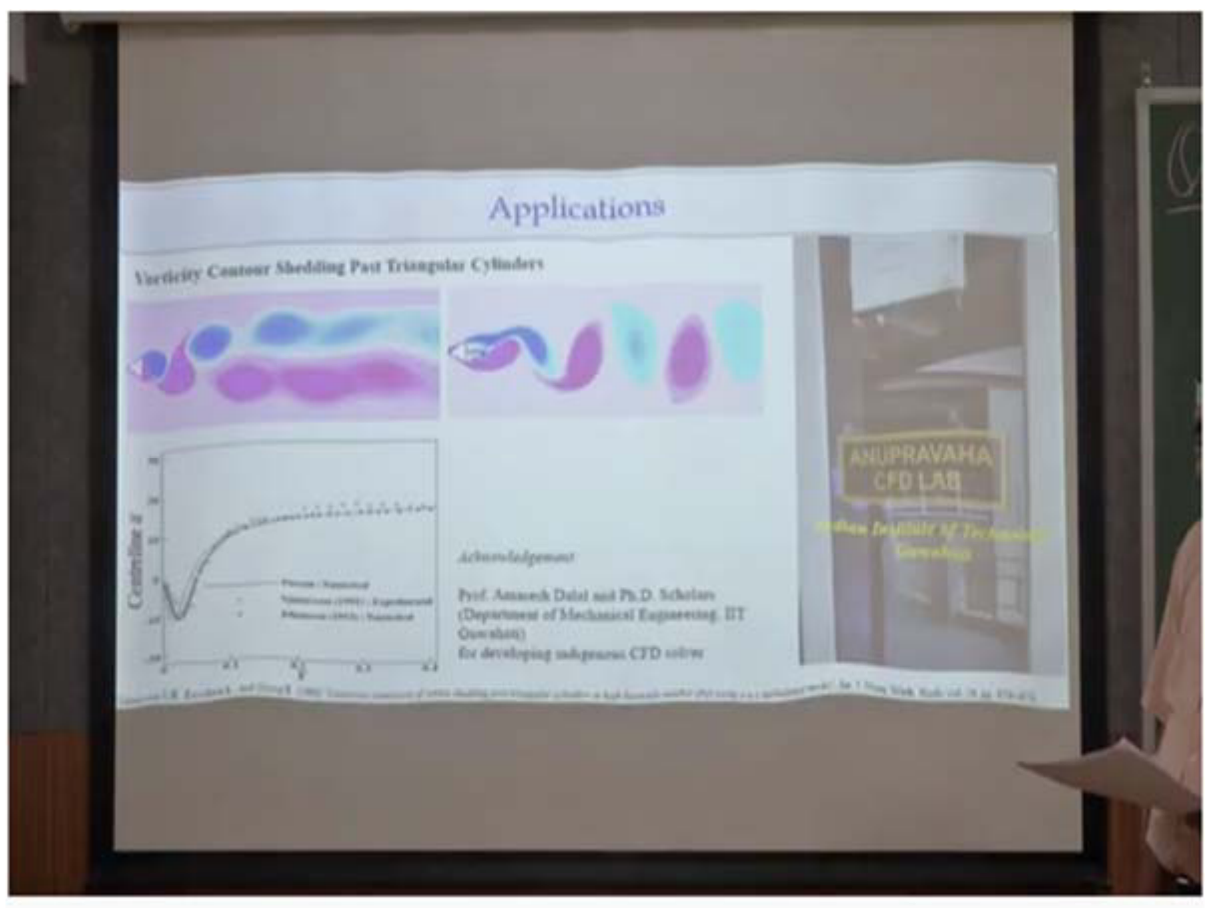

This is what the examples what earlier also we showed to you that if you have a triangular structures and how the way vortex sheddings are happening and it is quite interesting way if you see this unsteady patterns of waves vortex pattern. These are details obtained from the CFD solutions. So we can get it from the CFD solutions these type of vortex patterns and very interesting vortex patterns and that is what it shows that how these the average velocity line changes with that and with a comparison with experimental data.

So what I am to say that if you look at this advanced level of flow visualization technique and the experiment techniques what was available today we can have a very interesting flow problems. We can get the solutions. We can visualize the streamline, streaks line, the pressure distributions, the velocity distributions, the acceleration distributions all field we can see it. So with having these introductions levels, let us solve the 6 problems on the black.

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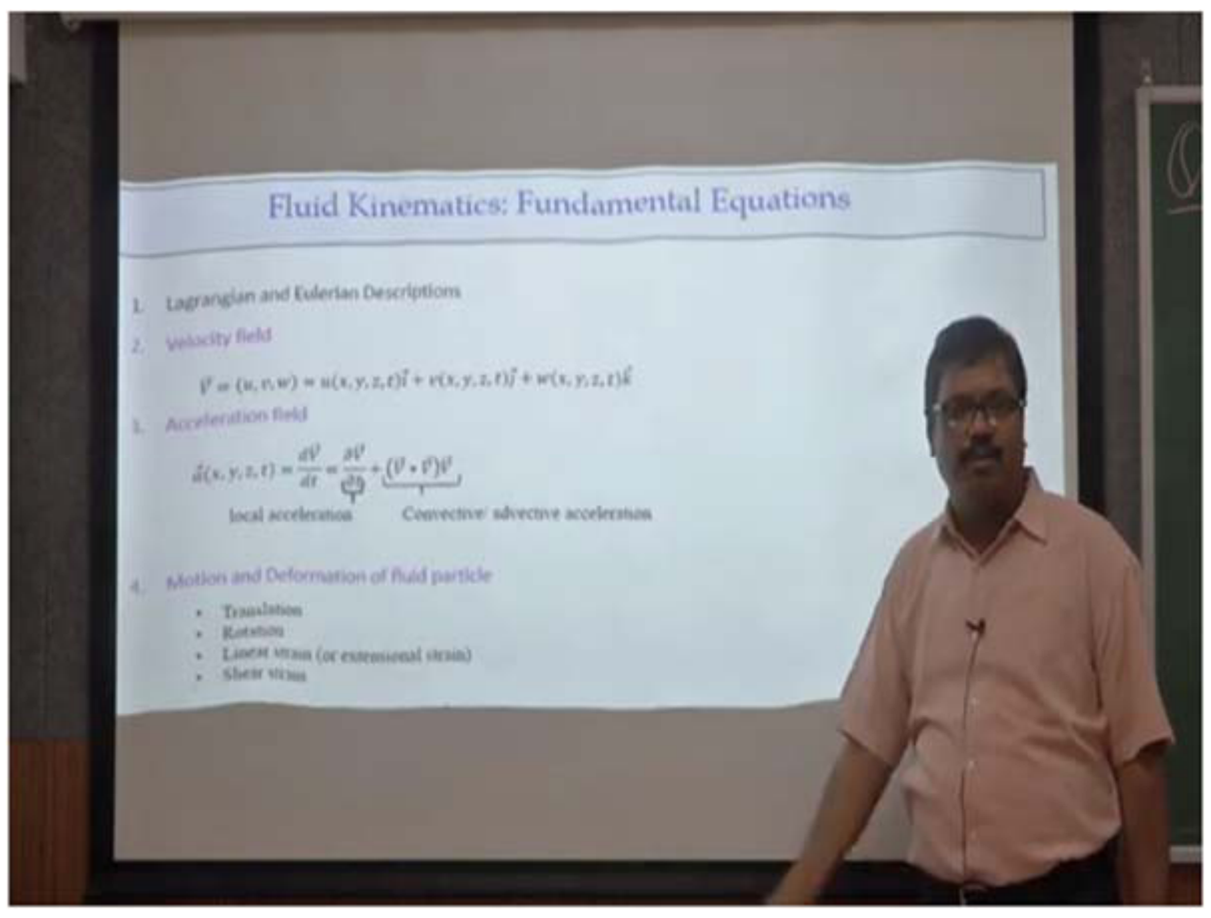

Before starting solving the blackboard applications, let me just have a recap that what we already discussed in the fluid kinematics is that you know that any velocity field we can define as 3 scalar components. The scalar component can have whether velocity scalar component can have whether positions and the time the independent part that is what the velocity distribution. Similar way the rate of the change of the velocity is accelerations but in terms of local accelerations and convective part we can define the acceleration terms.

Similar way when you have a motion of the fluid particles can have 4 type of conditions the motion and deformations like translations, rotations, linear strain, and the shear strain. We discussed more detail in the last class. So here I am just doing the recap for you to just look it this is the velocity distributions, the accelerations field and there could be the translations, rotations and the deformations like linear and shear strain.

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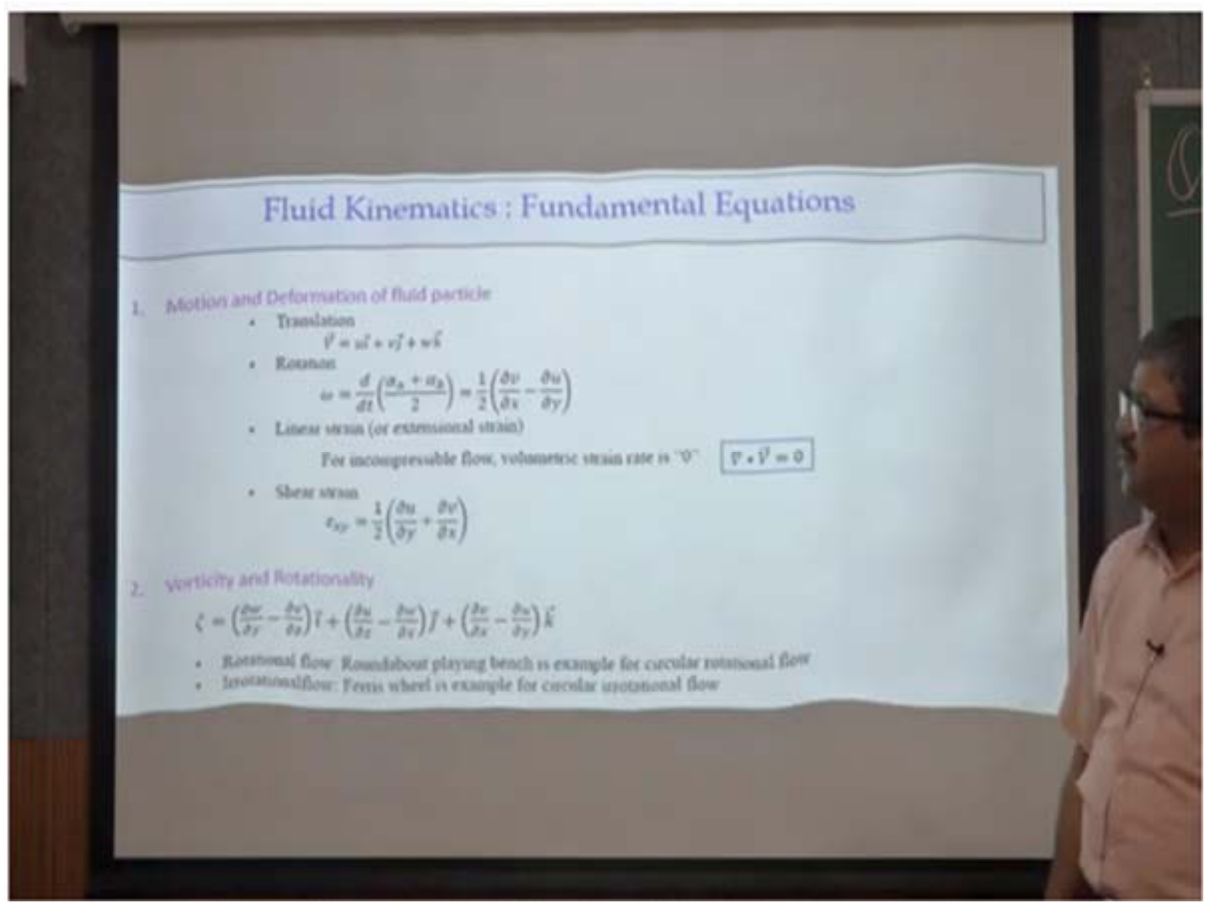

And if you look it if you have the rotations we can write the rotations quantity in terms of the velocity field. Similar way the shear strain components also we can write in terms of the velocity gradients and we have the vorticity measures what we derived very details we can compute the what could be the vorticity in different place. Look at that when you have the flow is incompressible flow that means when you have a volumetric strain equal to 0.

The delta dot product of the v velocity should be equal to 0. These the equations we were going to use more detail. When you are try to solve these simple problems what we are going to address on the blackboards. So the basically these are the recap the basic equations what we are going to use to solve the problems on the black.

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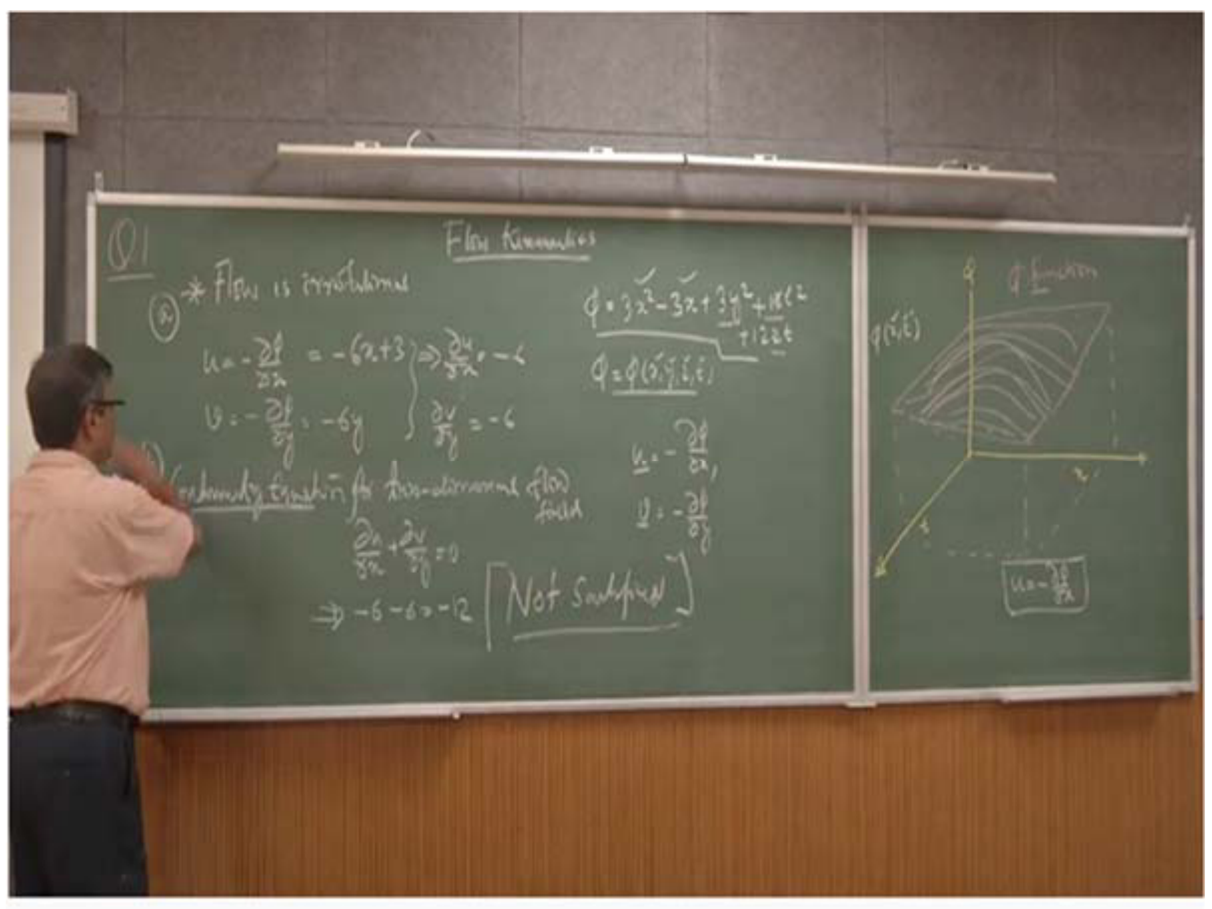

Let us start the first example 1.

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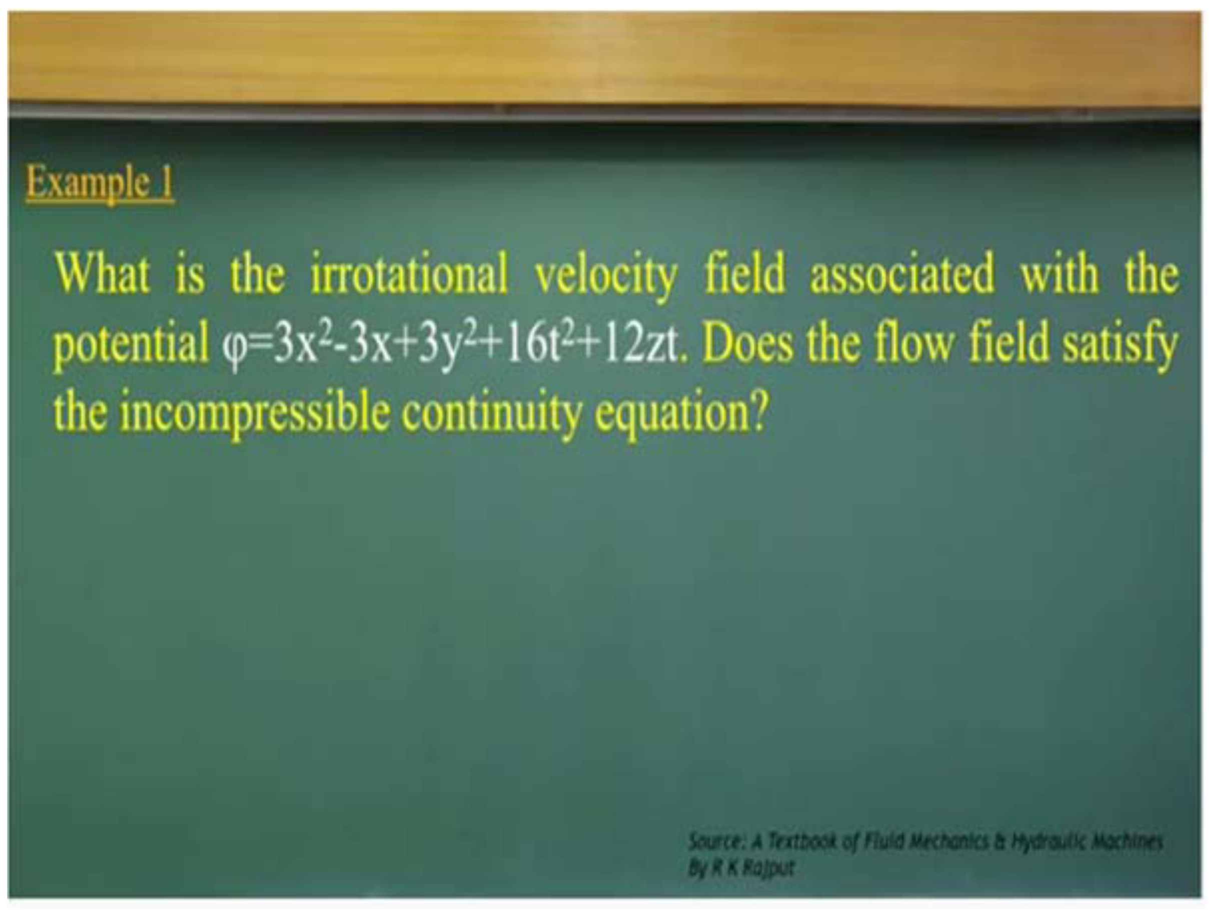

Example 1 says that what is the irrotational velocity field associated with the potential functions as given here. We have to find out the irrotational velocity field, does this flow field satisfy incompressible continuity equations? So it has 2 steps, the first steps were to find out what is the irrotational velocity field? So that is the points the already is given the flow is irrotational.

So that is the condition is given that means for this velocity potential functions which is the functions of if I write it is a function of x, y, z and the t the velocity potential function is the functions of the positions x, y, z and the t. If you can see these independent variables. So if you have the velocity potential functions which is function of x, y, z and t. Let me I just give a simple examples if I have the velocity potential function is a only 1 space functions and another time functions.

$$\varphi = 3x^2 - 3x + 3y^2 + 16t^2 + 12zt$$
$$\varphi = \varphi(x, y, z, t)$$

And if I plot this the functions could variables like this which is a functions of x and the t and to determine what could be the velocity? As the velocity potential function satisfy for this flow field we know from the definitions that the u will be a partial derivative of velocity potential functions and the v will be the partial derivative functions with respect to the y directions.

So these are the scalar component of v, u and the v the velocity functions what we can get it. So that is means if you look it the graphically what it indicates is that when you try to find out the velocity that means from the partial derivative directions the φ changes only the x directions when you do not consider other component that is what is of the negative of that is what is indicate for us the velocity field.

So looking that definitions let us come back to the problems which are quite easy problem for us to solve here that as we knew it the u is a the partial derivative of the φ with respect to x with negative sign because it is what it directions of that similar way the φ will have the negative signs of this. If I substitute that value that means if I just compute it that could be the partial derivative of φ with respect to x.

You can look it except these 2 terms I do not have a x term in here. So those terms becomes 0. So if you look it that way if you do a partial derivative of this I will get it - 6x + 3, same way if I compute it the scalar velocity field in the u, y directions which will be the partial derivative of with respect to the y. If you look at these equations again what we can see it in these equations only the y is in this term.

$$u = -\frac{\partial \varphi}{\partial x} = -6x + 3$$
$$\frac{\partial u}{\partial x} = -6$$

$$v = -\frac{\partial \varphi}{\partial y} = -6y$$

$$\frac{\partial v}{\partial x} = -6$$

So others are can be considered as a constants. So if you look at that and do a partial derivative φ with respect to the y, you will get -6y. So these are 2 velocity field which we obtain from this case. Now if we look at it these are the velocity field that is what is the part number a what we are looking for irrotational field. Now the second part of the problem does the flow field satisfy the incompressible continuity equations.

Since the flow field what we are looking in a 2 dimensional the continuity equations for 2 dimensional flow field will have

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0$$

. So it is a quite interesting to look at that means we can again do a partial derivative with respect to 1 x. This is what also partial derivative with respect to y and substitute it. If it is satisfied it then we can say it satisfy the continuity equations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0$$

$$-6 - 6 = -12 \quad \rightarrow not\ satisfied.$$

So if you look in this part if you do y del y del x then we will get it -6. Similar way del v/del y will also get -6. So substituting this value what will get it -6 -6 is -12. That means it is not satisfied. That is what is the answer for the second part okay.

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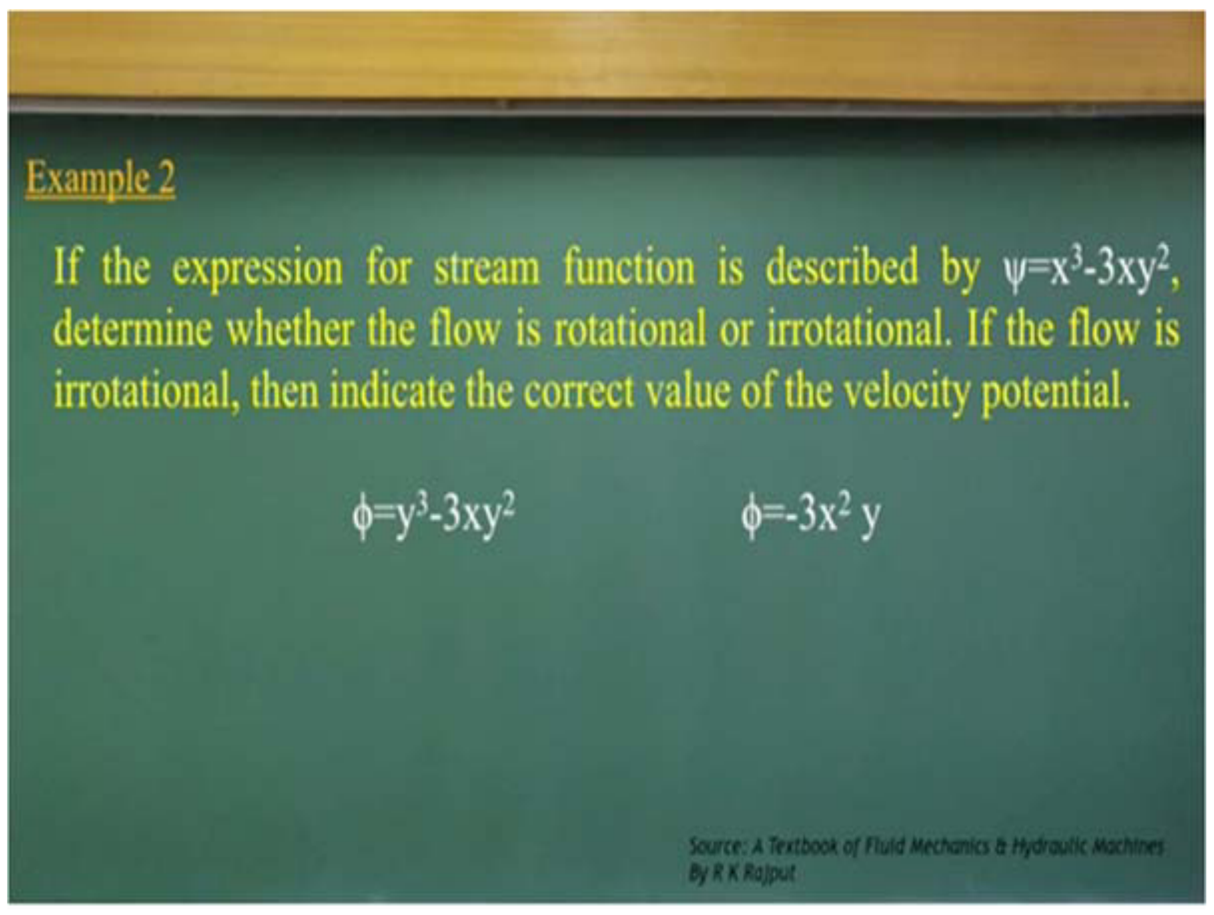

Let us solve this next example 2, in fluids the stream function is described at the Ψ functions.

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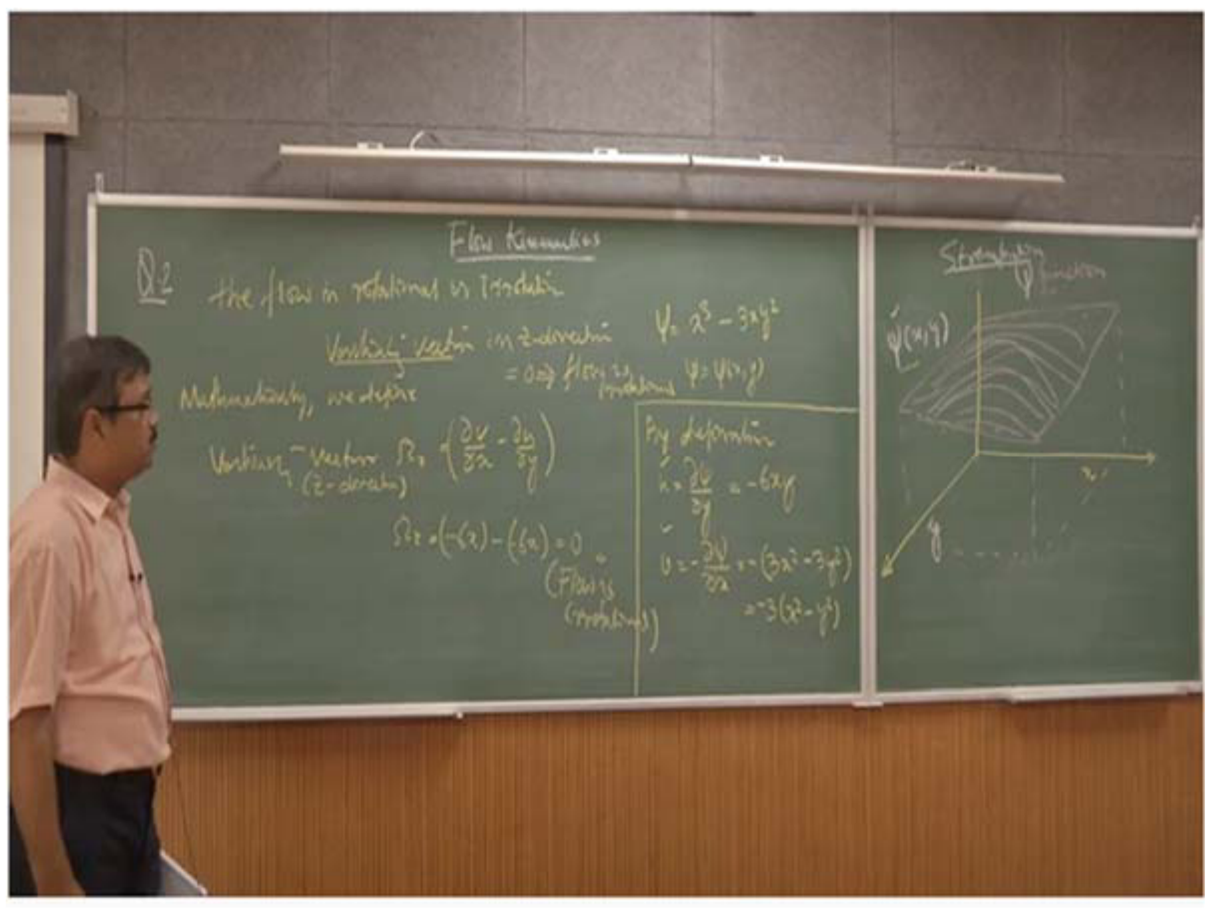

The stream functions as defined as

$$\Psi = x^3 - 3xy^2$$

determine the flow whether the flow is rotational or irrotational. If the flow is irrotational then indicate the correct value of velocity potentials. So the first part of the problem is that we have to prove whether the flow is irrotational or rotational. If again if I try to explaining you that now we have the stream functions which is functions of only the 2 dimensional x and y that is what could be a representing like a stream functions like this.

$$\Psi = \Psi(x, y)$$

And we try to find out whether the flow is rotational or irrotational. The questions comes is that to find out whether the flow is rotational or irrotational. What do you mean by that? That means we have to compute it the vorticity. If this vorticity becomes 0 that means we can say the flow is irrotational. So we have to find out the vorticity vector in this case it is only the 2 dimensional we need to have compute the vorticity vector in z directions okay.

$$\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

That means we need a vorticity vector of z direction components if that becomes 0 then that what indicate is the flow is irrotational that is the **fact** if not the flow is rotational. Let us compute the vorticity vector, the mathematically as we define the vorticity vectors as in the z directions is a functions of so it is a partial derivative of v scalar component with respect to y, scalar component of u with respect to y.

$$\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{k}$$

This is the vorticity vector component in the z direction. So we are just looking it in the z direction vector vorticity components and only z directions. Now the problem is very simple that will do a partial derivative of the stream functions to compute it what will be the v and u once you know that we can compute it what will be the vorticity vectors. What is the relationship between the stream functions and the velocity scalar velocity components as by definitions?

What we know it the velocity component of u will be the partial derivative with respect to y directions? If you look at this u components will have either the partial derivative of stream functions y directions that what is give me the scalar component in x directions and the v is given is the partial derivative with x directions. The negative of that it is represent the v component that means the scalar component in the y directions.

$$u = \frac{\partial \Psi}{\partial y} = -6xy$$

$$v = -\frac{\partial \Psi}{\partial v} = -3x^2 - 3y^2 = -3(x^2 - y^2)$$

By substituting this value of the stream functions we can get it this is what $-6xy$ and this is what minus of 3x square 3y square that is what will come out to be $-3(x^2 - y^2)$. Now as I know these u and the v component. I will just substitute here to compute it what is the vorticity vector in the z direction. This is very simple once I know the scalar component of even v which is a function of the partial derivative of the stream functions I once I get that things that what I can substitute here to compute it.

$$\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\zeta = -6x - (-6x) = 0 \rightarrow \quad flow\ is\ irrotational$$

That is what you can compute do partial derivatives where we are not going a step by steps but what you can get it - 6 x minus of – 6 x that is equal to 0. That is what indicate this flow is irrotational that means that is what is the first part of the component is that we have proved that the flow is irrotational as the vorticity vector becomes 0 that the first components.

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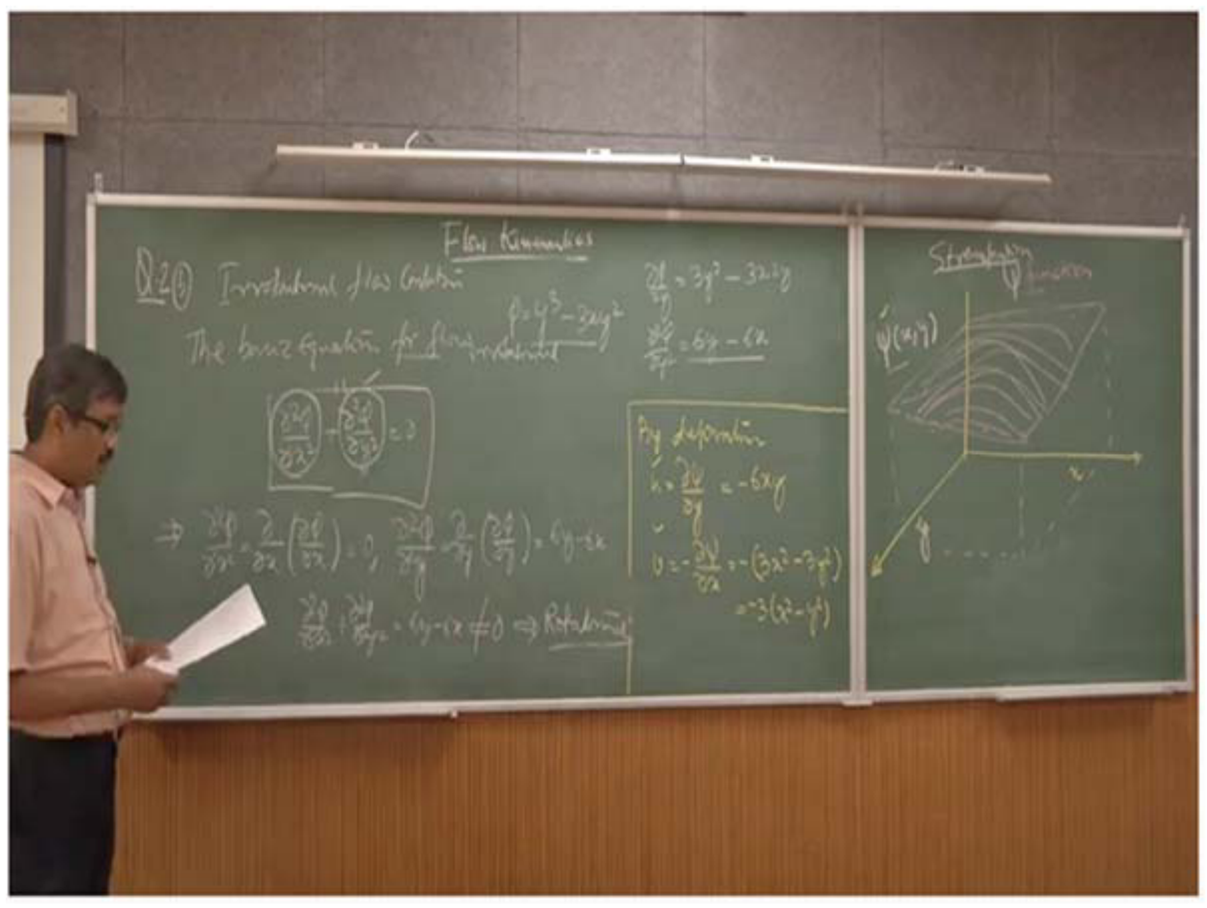

So the second part of the component is that if the velocity potential functions is given to us okay the velocity potential function is given to us in these case we have

$$\varphi = y^3 - 3xy^2$$

We need to find out whether this is what also indicate it is a flow is irrotational that means the condition is irrotational flow conditions. That is what also we can get it the basic equations is for flow irrotational is the Laplace equations with respect to the velocity potential functions.

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=0$$

That means as it is equal to 0. This is the Laplace functions with respect to the φ in the 2 dimensional form, the x and the y components. So that is what is indicating for us to substitute this the phi value and try to look it does it satisfy is equal to 0. If it is equal to 0 then flow is irrotational. When I substitute these value that means we can put the first components is that the which is if you just split like this you can easily find out what could be the values.

Like for examples if you put it these function

$$\frac{\partial\varphi}{\partial y}=3y^2-3x2y$$

$$\frac{\partial^2\varphi}{\partial y^2}=6y-6x$$

We can split like this and looking these terms, you can find out the first partial derivative will give us -3y square and second one if I do it which does not have a function of x then definitely it will give us the 0 value.

The same way if I compute the partial derivatives of the with respect to y that is what will give you the 6y - 6x.

$$\frac{\partial^2\varphi}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x}\right)=0$$

$$\frac{\partial^2\varphi}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial\varphi}{\partial y}\right)=6y-6x$$

So if I substitute these values the Laplace equations of phi x square here that what will get it 6y - 6x. So it is not equal to 0.

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=6y-6x\neq 0$$

That means the flow is rotational and it does not satisfy the basic equations of the Laplace equation which is a continuity equation for the profit. The similar way if I go for the second components where another phi function is given.

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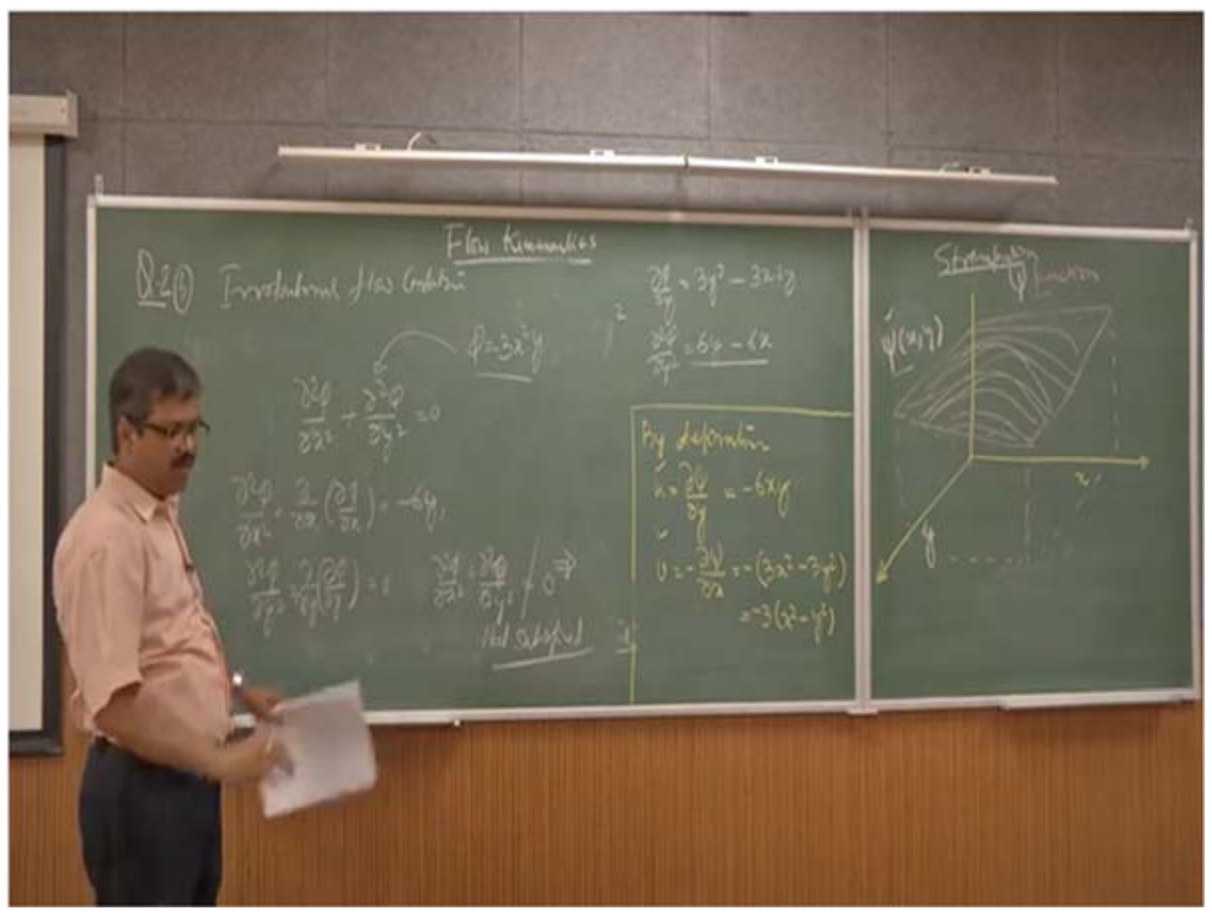

$$\varphi = -3x^2y$$

So were to again just change the functions find out whether it satisfy the Laplace equations of the phi value. If it satisfy with the Laplace equations of the phi value then we can say that this is for the velocity potential functions for this process. If I substitute these once again like to compute the partial derivative of φ with x in a second partial derivative of the φ with respect to x that what will give us if I substitute these values that is what will give this for here is -6y.

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x}\right) = -6y$$

Similar way if I look it since it is only the first order y is there I can easily say it this what will becomes 0.

$$\frac{\partial^2\varphi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial\varphi}{\partial y}\right) = 0$$

So if you look it this 2 term -6y and 0 definitely the Laplace equations after substituting this Laplace equations that what will not be equal to 0, so it is not satisfied.

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}=-6y\neq 0$$

This is what very simple examples just to look it by substituting this the velocity potential function that is satisfy the Laplace equations.

If satisfy the Laplace equations that means it is the velocity potential field. If not, then it is not a appropriate velocity field to define this flow of behaviours. So that what we have to look it for these keys.

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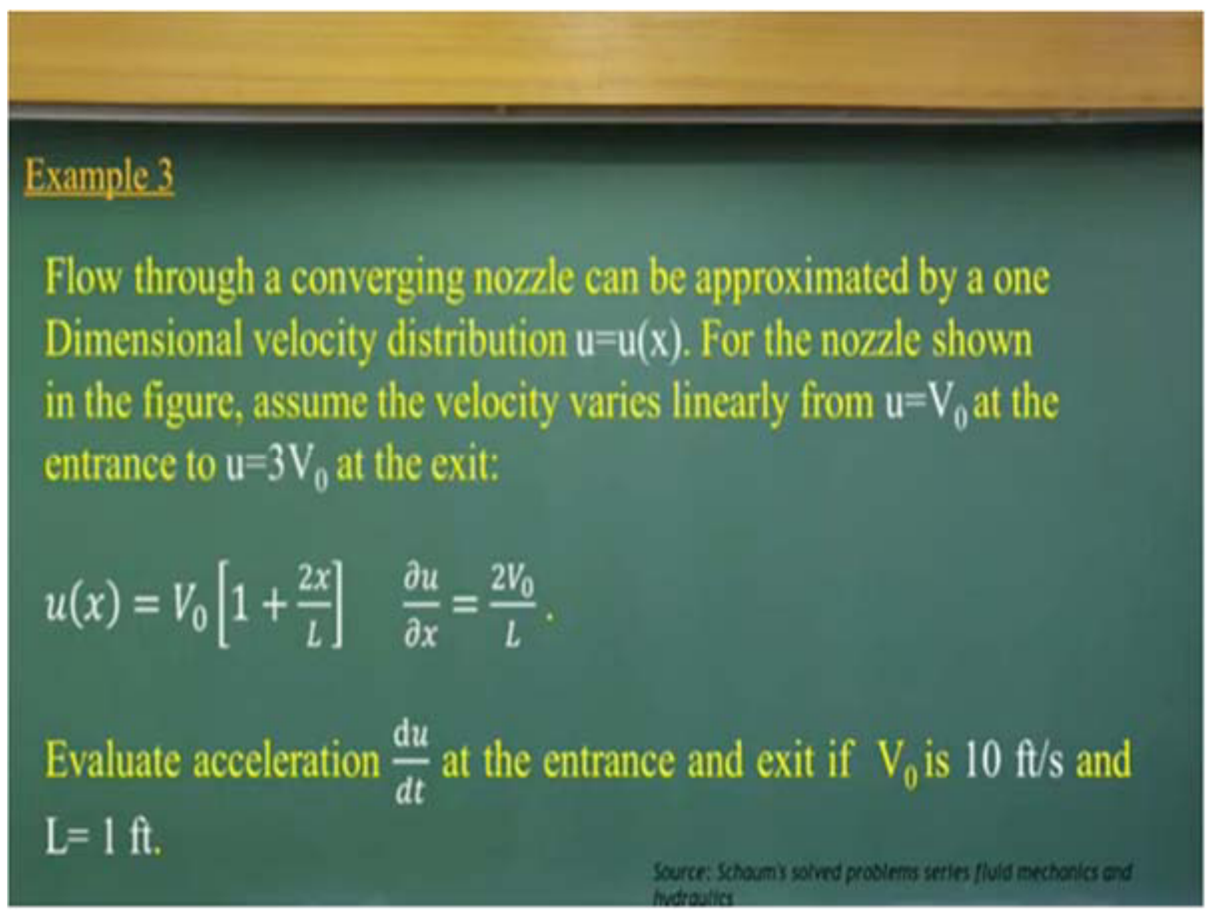

Let us solve the third examples which is a very interesting problems. The flow through a converging nozzle as given in the figure can be approximated as 1 dimensional flow velocity distributions $u = u(x)$ and from the nozzle shown in this figure and assume the velocity varies linearly at this point is $u = V_0$.

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